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Book Review: *Path Integrals*

Path Integrals. Edited by G. J. Papadopoulos and J. T. Devreese. Plenum, New York, 1978, x+515 pp.

Path integration is one of those topics in theoretical physics which never became quite fashionable, yet never went out of grace either. Path integrals form a powerful instrument for looking at problems from a new point of view and for developing new approximation methods. The development of this topic through 1955 forms the subject of the well-known book by Feynman and Hibbs.[1] The first international conference[2] was held in London in 1974. The present volume, which contains the lecture notes of a NATO advanced study institute held in Antwerpen in 1977, reports on progress made since about 1970. As most quantities characteristic of many-body systems can be represented by path integrals, the applications cover a large part of theoretical physics. The sixteen contributions to this volume can be naturally divided into those in which path integrals are used to derive exact results and those devoted to approximation methods for their evaluation.

The volume opens with general introductory lectures by Thornber, Klauder, and Papadopoulos. The chapter by the latter author contains an interesting historical survey and a list of references which is almost complete up to 1975. Exact representations of the partition function as a path integral are discussed for Bose systems by Papadopoulos and by Wiegel and for Fermi systems by Papadopoulos and by Sherrington. In the Fermi case the "paths" are elements of an anticommuting Grassman algebra.

Luttinger shows how path integrals can be used to derive both upper and lower bounds on the partition function of a disordered system. His method also gives a lower bound on the ground-state energy of a quantum system. As the usual variational principle provides an upper bound, this enables one to bracket the ground state. The reader is warned of a misprint in Eq. (2.23) of this chapter, which should display a $\geqslant$ sign.

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[1] R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965).

[2] A. M. Arthurs, ed., *Functional Integration and Its Applications* (Clarendon Press, Oxford, 1975).

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The main approximation method for path integrals is an adaptation of the saddle point method to integrals of functions of infinitely many variables. It is impressive to see throughout the book how much further this method has been developed since its naive form reported in the book by Feynman and Hibbs. For example, Gutzwiller uses this method to quantize ergodic systems and shows that the semiclassical density of states is related to a sum over all classical periodic orbitals weighted with the appropriate complex phase factors. His discussion of an almost forgotten paper by Einstein[3] on the quantization of ergodic systems is unusually fascinating. In the chapter on the interacting Bose fluid it is shown that the saddle point method leads in a natural way to a description of the lambda transition in terms of quantized vortex lines. This chapter also makes contact with current work on the renormalization group approach. In one of the seminars which accompany the main lectures Levit discusses the semiclassical approximation to path integrals from the point of view of caustics and catastrophies. Here, too, the plurality of the classical trajectories plays an essential role. It seems to me that we are only just beginning to understand the mysteries of saddle point asymptotics in function space and the wealth of physical phenomena which it engenders.

Mühlschlegel reviews the path integral representation of the Fokker–Planck equation. This approach originated with two papers by Wiegel[4] in which older results by Onsager and Machlup[5] and Falkoff[6] were generalized to the nonlinear regime. In recent years this topic has generated a considerable literature because a certain ambiguity in the definition of the path integral for the propagator leads to different expressions for the Lagrangian. It seems this ambiguity has been clarified now through the work of several authors, and applications of the path integral to the explicit calculation of transition probabilities are forthcoming.

Macromolecules are the incarnation of path integrals. Applications to the statistical mechanics of macromolecules are the subject of a contribution by Edwards. Here the treatment of the mutual topological entanglement of a polymer and a straight line, first solved by Prager and Frisch[7] using other methods, is particularly pretty, and a true terra incognita opens itself to the reader. Some remarkable Russian work[8] is not mentioned, however.

Nonequilibrium theory is discussed by Hosokawa. This author rewrites

[3] A. Einstein, *Verhandlungen der Deutschen Physikalischen Gesellschaft* **19**:82 (1917).

[4] F. W. Wiegel, *Physica* **33**:734 (1967); **37**:105 (1967).

[5] L. Onsager and S. Machlup, *Phys. Rev.* **91**:1505 (1953).

[6] D. Falkoff, *Ann. Phys. (N.Y.)* **4**:325 (1958).

[7] S. Prager and H. L. Frisch, *J. Chem. Phys.* **46**:1475 (1967).

[8] M. D. Frank-Kamenetskii, A. V. Lukashin, and A. V. Vologodskii, *Nature* **258**:398 (1975).

the BBGKY hierarchy in the form of a single functional equation, from which the irreversible dynamics is obtained in an elegant but somewhat ad hoc fashion.

An important part of this volume is concerned with applications to solid-state theory. As this field is outside the competence of the reviewer, I will only quote the authors and their topics: Devreese (physics of polarons), Sherrington (magnetic alloys), Thornber (dissipative systems), Mühlschlegel (Anderson model), and Nettel (piezoelectric semiconductors).

A fitting epilogue to the conference is the chapter written by Rosen on the use of path integrals in field theory, especially in the search for classical solutions to the field equations. It seems that in this case field theorists are developing methods which have already been explored to a certain extent in many-body theory. It is to be hoped that a future conference on this topic will lead to a merger of these two lines of development.

The book is recommended as a timely collection of review papers reflecting the state of the art of path integration; it will also be of great help to pick the most promising lines of further research in this field.

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